Classical Problems of the Turbulent Boundary Layer

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Abstract: Traditional concept about the boundary layer structure, technique of the theoretical determination of a velocity profile in this layer, developed in first half of the 20th century, does not experienced, basically, any changes, although, on their close analysis a number of questions and even problems arise requiring their solution.

First of all, it is referred to physical causes of the occurrence of turbulence in the boundary layer, conservation of its near-wall region, conformance of a classical logarithmic velocity profile to experimental data, the appropriateness of introduction of such definition as “a laminar sublayer in the turbulent boundary layer”. All these outstanding problems are considered in the following materials.

Keywords: turbulence, boundary layer, laminar sublayer, “floating” coordinates, deformation conditions, angular velocity.

1. Introduction

While the integral calculation methods were used, which always were controlled by the physical experiments at the relevant models upon development of new units, the issues concerning the boundary layer structure and the physical model of the laminar flow destruction in this layer were to a great extent of the theoretical interest.

Situation was fundamentally changed due to the rapid development of the numerical calculation methods of complex aerodynamic challenges, and there was an illusion of the possibilities of calculation of flow fields without division of their fields into zones mentioned by L. Prandtl. The confidence in accuracy of such calculations was proved to be so high and the calculation cost was so lower that a physical simulation of the real objects was replaced quickly by a mathematic simulation and this confidence, basically, has stopped development of the theoretical fluid and gas dynamics.

However, wide and common usage of the continuous calculation methods of the velocities and pressures fields allowed identification of several tasks, where the calculation results did not comply with known experimental data and physical flow patterns.

In general, nonconformity of calculation with experimental data was found in those tasks where a flow pattern in the boundary layer had impacted crucially the whole flows field.

This circumstance has created several clarifying theories of the turbulence that are used to close the incomplete Reynolds equation system being the basis for all existed calculation programs. However, in such a way the flow features in the boundary layer can hardly be accounted.

Accordingly, it would be desirable to consider the boundary layer structure and impact of the Reynolds number on this structure, and those concepts about a flow pattern in the boundary layer which came up almost 100 years ago.

2. Mechanical model of turbulence occurrence

In case of formation of turbulent flows the link between the occurring additional stresses in the turbulent flow and the averaged velocity components is a fundamental task of the fluid and gas dynamics. All classical and many new turbulence theories are dedicated to determination of this link.

At that, physical mechanism of the turbulence occurrence remains, basically, discreet.

This situation is most clearly reflected in papers of G.V. Shubauer and M. Chen [4], and can be summarized by saying that:

1. Mechanism of transition from the laminar to turbulence flow is not outside, but inside of the flow itself.
2. The turbulence occurrence is spontaneous.
3. The presence of shear flows is a necessary condition of the turbulence occurrence. [3]

On the basis of given conclusions there are all grounds to consider that the reason for existence of two shapes of the flow in fluids and gases lays in the capacity of examined media to have a small deformation of their “fluid” elements.

Such property has been reflected in known Helmholtz theorem, according to which (in contrast with solid bodies) the motion of fluid, in general, consists of not two, but three motions – translational, rotational and deformation motion. (The last is absent in the solid body motion).

In the case of plain flow the translational motion is determined by a velocity vector \( \mathbf{v} \), the rotational motion is characterised by an angular velocity \( \omega \), and the deformation motion of a plain fluid element is determined by an angular deformations velocity \( \delta \).

In the limits of the boundary layer, it may be assumed (with a small inaccuracy) that \( \omega \) and \( \delta \) are equal to \( \omega_z \), accordingly.

Change in a transverse velocity gradient in any section of the boundary layer causes a change in angular deformation velocity \( \delta \) of a common fluid element. In this case, due to deformation motion the rotation center of the considered fluid element attempts to maintain its primary position, and the center of an elementary mass shifts relatively to the rotation center by a value \( e \).

Schematically this process, in respect to a plain “fluid” element, is illustrated in Fig. 1.
\[ dR = dm \cdot \omega^2 e \]  
(1)

At small velocities of an angular deformation \( \delta z \) (at small transverse velocity gradient \( du/\delta y \)) the originating eccentricity \( e \) is eliminated due to viscosity forces, and natural “balance” of the rotating elementary fluid particles takes place. In this regard, the laminar conditions represent a condition of the “balanced” flow. In case of the high angular deformations velocities of the elementary “fluid” volumes the occurred additional force \( dR \) can not be compensated by the molecular shear forces, and the concerned “fluid” element will be displaced from its fluid flow line capturing at that some portion of fluid situated inside a circle, which is described by the mass center (point \( b \) in Fig. 1) at its rotation about the rotation center (point \( a \) in Fig. 1). At plain flow, the fluid situated inside the circle of radius \( e \) forms the initial vortex core, which is rotating so far by the law of a solid body. (In the real 3-dimensional flow this is already the initial volumetric vortex core).

In case of a plain flow, an elementary fluid mass \( dm \), given in Fig. 1, will be equal to

\[ dm = \rho \cdot dx dy \cdot l \]  
(2)

Then, force \( dR = \rho \cdot dx dy \cdot l \cdot \omega^2 e \), and an additional force \( \tau \) will occur at the small areas paralleled to plane \( x-z \), which is equal to

\[ \tau = \frac{dR}{dx \cdot l} = \rho \cdot e \cdot dy \omega^2 z \]  
(3)

As \( \omega_z = \frac{1}{2} \left( \frac{du}{\delta y} \right) \), and assuming that a linear value \( dy = \text{const} \cdot e = l \), we formally obtained the Prandtl formula for turbulent forces in a shear flow

\[ \tau = \rho l^2 \left( \frac{du}{dy} \right)^2 \]  
(4)

In this case, linear dimension \( l = \text{const} \cdot e \) is a direct function of angular deformation velocity \( \delta z \) and, therefore, contrary to the “mixing length” \( l \) in the Prandtl formula in respect to the boundary layer, it reaches a maximum value near the wall and goes to zero at the outer edge of the mentioned layer.

3. The physical interpretation of conservation of the near-wall area of the turbulent boundary layer

The examined mechanism of turbulence generation allows to explain known experimental data according to which the near-wall part of the turbulent boundary layer, determining the interaction nature of moving media with streamlined surfaces, found to be conservative in relation to many external influences.

Thus, in [5] it is mentioned that “the turbulence of an external flow has no influence on processes occurring in so called area of production and dissipation of turbulence, and the transverse and longitudinal gradients of the pressure and surface curvature have no influence on them also”. (1956)

Velocity profiles, given in Fig. 2, obtained in the gradientless flow at three values of the turbulence number \( E0 \) in the approach flow, which are equal to \( E01 = 1.7\% \); \( E02 = 4.6\% \); \( E03 = 6.8\% \), clearly demonstrate the correctness of this statement.

Fig. 2: Velocity profiles at different turbulence values

It is apparent that in the near-wall area all experimental velocity values are grouped near one curve and then, due to increase in physical thickness of the boundary layer, the experiment points form three different profiles while the external turbulence increases.

Experimental data, given in Fig. 3, on distribution of relative shear forces \(- \) shear force on the wall) in the transverse section of the gradientless boundary layer, obtained in the rectangular channel of the uniform cross-section when the turbulence number \( E0 \) changes from 0.3% to 4.7%, is more evident. [7]

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Fig. 3: Distribution of relative shear forces in the transverse section of the gradientless boundary layer

In this case, a relative shear force value is not changing when the turbulence number changes at a distance from the wall equal to 40% of the boundary layer thickness.

If we will be based on the above mentioned mechanism of the turbulence generation according to which especially the near-wall area is an area of the intensive turbulence generation where directly near the wall, as per the experimental data [3], the turbulence numbers are reaching 8÷10%, then above given experimental data (Fig. 2 and Fig. 3) allows to assume that until disturbances from the side of an external flow will exceed 8÷10% the near-wall part of the turbulence layer shields reliably the streamlined surface from mentioned external disturbances.

4. The laminar sublayer – favorable theory or physical reality

Idea about the laminar sublayer, introduced to make a physical meaning of the logarithmic velocity profile, happened to be a “fruitful” theory, and in the most literary sources there are no doubts in this regard.

Papers [2,3,4,9] expressly stated about the impossibility of existence of velocity pulsations in the laminar sublayer and, therefore, on the wall. In other words, the laminar sublayer provides near the wall the layered flow pattern without velocity pulsations and, thus, pressures.
However, direct pressure measurements made directly on the walls of the plane and axisymmetrical channels of a uniform cross-section always fix the significantly large amplitudes of pressure pulsations. Apart of this, probability of existence of the laminar sublayer itself creates the several questions without response.

First, according to the concerned double-level structural model, there should be no pulsation components of velocity and pressure in the laminar sublayer as if they exist, the relation associating a molecular shear force \( \tau = \mu \frac{\partial u}{\partial y} \) (where \( \mu \) - viscosity factor), which is used for determination of velocity \( u \), on the outer edge of the laminar sublayer, cannot be used.

However, as it follows from [3] and our experimental data, the significantly large pulsations of pressure and as well as velocity are fixed in close proximity from the wall and on the wall itself. Thus, a turbulent shear in the near-wall area cannot be ignored and here \( \tau = \mu \frac{\partial u}{\partial y} + (-\rho \mu \nu) \).

As a result of this, if there are turbulent shear forces in the laminar sublayer, the binding edge line of the conventional laminar sublayer with the remaining (turbulent) part of the boundary layer cannot be accurately determined.

Second, when the laminar sublayer conjugates a turbulent part of the logarithmic velocity profile, dynamic velocities \( V_t \) in jointed layers are taken equal. In other words, \( V_t = V_n \).

As \( V^2_t = \omega^2_t y^2 \left( \frac{du}{dy} \right)^2 \) (formula 2), and in the laminar sublayer:

\[ V^2_n = \nu \frac{du}{dy} \]

then, from equation of mentioned dynamic velocities it follows that

\[ \nu = \omega^2_t y^2 \frac{du}{dy} \]

Meaning that a physical constant – kinematic viscosity – becomes a position function of the corresponding section in the boundary layer. Finally, when the static pressures are measured directly on the wall using quick-response transducers, the pressure pulsations, typical (by frequency amplitudes) especially for the near-wall turbulence, are always fixed at transition to the turbulent flow.

Expressed reasons disregard the hypothesis about physical existence of the laminar sublayer rather than support it.

5. Conclusion

It is shown that the demonstrated in the scientific literature, the brilliant coincidence of the logarithmic velocity profile in a cylindrical pipe in turbulent flow with the experimental data is the result of the use of “floating” coordinates, changing their maximum values when changing the Re numbers, resulting in matching at the specified coordinates of the experimental point in a fixed coordinate system generally belong to different (height) cross-sections of the boundary layer.

First formulated and proved in experiments the position of the shielding properties of a turbulent boundary layer, which ensures the conservativeness of the local coefficients of resistance with respect to the degree of turbulence outside the boundary layer.

6. Literature